A Perturbation Technique for the Finite Element Modelling of Differential Probes in Nondestructive Eddy Current Testing

Ruth V. Sabariego and Patrick Dular

Dept. of Electrical Engineering and Computer Science (ELAP), Institut Montefiore

University of Liège, Sart Tilman Campus, Grand Traverse 10 (B28), B-4000 Liège, Belgium

R.Sabariego@ulg.ac.be, Patrick.Dular@ulg.ac.be

Abstract

The present paper deals with a new finite element scheme for nondestructive eddy-current testing (ECT) problems involving multiply connected test pieces and differential probes. It concerns a perturbation technique applied to the magnetodynamic $\boldsymbol{h} - \phi$ formulation.

The unperturbed field (in the absence of the flaw) is conventionally computed in the complete domain. The source of the perturbation problem is then determined by the projection of the unperturbed field in a relatively small region around the defect, the optimum size of which depends on the working frequency. The discretisation of this reduced domain is well adapted to the size of the defect and chosen independently of the dimensions of the excitation probe and the specimen under study. At a discrete level, the voltage change is efficiently computed by integrating only over the defect and a layer of elements in the reduced domain that touches its boundary.

The accuracy of the proposed perturbation model is evidenced by comparing the results obtained for different dimensions of the reduced domain to those achieved in the conventional way. The considered test case involves a differential probe scanning the outer surface of a metal tube for the detection of perforating cracks.

1 Introduction

The ultimate goal of nondestructive eddy-current testing (ECT) is to determine the position and size of defects in conducting materials (inverse problem). However, a fast and accurate calculation of the probe response (forward problem) is often required for identifying the flaws from measured data.

Several variants of the volume integral method (VIM) have been reported in literature [1], [3] for solving this kind of problems. Herein, defects can be represented by a distribution of current dipoles in its volume. A boundary element method, using VIM for describing the defect, is proposed in [4]. Only the crack is discretised. The calculations associated to different probe positions are very fast. Nevertheless, these techniques become extremely expensive in case of more complicated geometries (other than infinite stabs or tubes with homogeneous and linear material parameters). Another disadvantage of the method is the numerical complication due to xthe singularity of the Green's kernel.

The finite element method [2] allows to overcome these drawbacks. However, it may require a dense discretisation in the vicinity of the defect resulting in a large 3D mesh. The impedance change due to the defect is calculated as the difference of the impedance values with and without flaw. Further, calculations for different probe positions are performed independently, which is time consuming.

A $\boldsymbol{h} - \phi$ formulation based finite element scheme that calculates directly the distortion of the eddy currents due to a flaw was proposed by the authors in [5]. The counterpart $\boldsymbol{b}$-conform formulation is treated in [6].

Herein, the computation is split into a computation without flaw and a computation of the field distortion due to its presence. The unperturbed field is calculated in a large region taking advantage of any symmetry or analytical solution, and applied as a source in the flaw for the second computation. The perturbed field can thus be determined in a reduced domain around the defect, what allows for a discretisation which is better adapted to the size of the defect.

This paper deals with an extension of the method for taking into account multiply connected test pieces and differential probes. As test case, a tube with a crack and a differential probe that scans its outer surface is considered.

2 Perturbation method

We consider a magnetodynamic problem in a bounded domain Ω (boundary Γ) of $\mathbb{R}^3$. The eddy current conducting part of Ω is denoted Ω_c and the non-conducting one Ω_c^C ($\Omega = \Omega_c \cup \Omega_c^C$). Source conductors, with a given current density $\boldsymbol{j}_s$, are comprised in $\Omega_s \subset \Omega_c^C$. A flaw Ω_f (boundary Γ_f) appears in Ω_c. The source term is obtained from the eddy-current distribution without defect of the unperturbed formulation.

For the sake of simplicity, let us assume hereafter a zero-conductivity flaw $\sigma_f = 0$ with the same magnetic permeability $\mu_f = \mu$ as the host material Ω_c. Further both are linear and isotropic as well. Let us particularise the Ampère law for the unflawed (subscript u) and flawed (subscript f) arrangements, i.e. curl $\boldsymbol{h}_u = \sigma \boldsymbol{e}_u$ and curl $\boldsymbol{h}_f = 0$, where $\boldsymbol{h}$ and $\boldsymbol{e}$ are the magnetic and electric fields. Subtracting these two expressions, the source generating the perturbation $\boldsymbol{j}_{sf}$ is an electric current density in the flaw given by

$$\boldsymbol{j}_{sf} = \text{curl}\, \boldsymbol{h} = -\sigma \boldsymbol{e}_u = -\text{curl}\, \boldsymbol{h}_u \,, \tag{1}$$

with $\boldsymbol{h} = \boldsymbol{h}_f - \boldsymbol{h}_u$ the perturbation magnetic field [5].

2.1 $\boldsymbol{h} - \phi$ magnetodynamic formulation

Adopting the magnetic field formulation, the general expression of the magnetic field $\boldsymbol{h}$ in Ω is $\boldsymbol{h} = \boldsymbol{h}_s + \boldsymbol{h}_r$, with $\boldsymbol{h}_s$ a source field in Ω satisfying curl $\boldsymbol{h}_s = \boldsymbol{j}_s$ and $\boldsymbol{h}_r$ the reaction field in Ω_c. In the non-conducting region Ω_c^C, the reaction field $\boldsymbol{h}_r$ can be derived from a scalar potential ϕ such that $\boldsymbol{h}_r = -\text{grad}\, \phi$.

The $\boldsymbol{h} - \phi$ magnetodynamic formulation is obtained from the weak form of the Faraday law:

$$\partial_t(\mu \boldsymbol{h}, \boldsymbol{h}')_\Omega + (\sigma^{-1} \text{curl}\, \boldsymbol{h}, \text{curl}\, \boldsymbol{h}')_{\Omega_c} + \langle \boldsymbol{n} \times \boldsymbol{e}, \boldsymbol{h}' \rangle_\Gamma = 0 \,, \qquad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega) \tag{2}$$

where $\boldsymbol{n}$ is the outward unit normal vector on Γ, part of the boundary of Ω; $(\cdot,\cdot)_\Omega$ and $\langle \cdot,\cdot \rangle_\Gamma$ denote a volume integral in Ω and a surface integral on Γ of the product of their arguments; $F_{h\phi}(\Omega)$ is the function space defined on Ω and containing the basis functions for $\boldsymbol{h}$ (coupled to ϕ) as well as for the test function $\boldsymbol{h}'$ [7]. At the discrete level, this space is built with edge finite elements. The trace of $\boldsymbol{e}$ is a constraint associated with Γ (this constraint can e.g. be associated with a homogeneous natural boundary condition or with a global quantity) [7].

The unperturbed field $\boldsymbol{h}_u$ (with $\Omega_f \subset \Omega_c$) is obtained by particularising ($\boldsymbol{h} = \boldsymbol{h}_u$) and solving (2). This field $\boldsymbol{h}_u$ is then projected on a reduced domain $\Omega' \subset \Omega$ around the defect. Note that projecting only $\boldsymbol{h}_u$ is not sufficient as this way the local current $\boldsymbol{j}_u = \text{curl}\, \boldsymbol{h}_u$ will not be conserved. Furthermore, the trace of source field $\boldsymbol{n} \times \boldsymbol{h}_{sf}$ on Γ_f ($\boldsymbol{h}_{sf} = -\boldsymbol{h}_u$) contributes to the exterior domain $\Omega' \backslash \Omega_f$. Indeed, the following interface condition has to be satisfied on Γ_f

$$\boldsymbol{n} \cdot \boldsymbol{j} \mid_{\Gamma_f} = -\boldsymbol{n} \cdot \boldsymbol{j}_{sf} \mid_{\Gamma_f}, \tag{3}$$

which is equivalent to considering

$$\boldsymbol{n} \times \boldsymbol{h} \mid_{\Gamma_f} = -\boldsymbol{n} \times \text{grad}\, \phi \mid_{\Gamma_f} + \boldsymbol{n} \times \boldsymbol{h}_{sf} \mid_{\Gamma_f} . \tag{4}$$

The source of the perturbation problem in Ω_f is calculated through a projection method in Ω' as

$$(\text{curl}\, \boldsymbol{h}_{sf}, \text{curl}\, \boldsymbol{h}')_{\Omega'} - (\boldsymbol{j}_{sf}, \text{curl}\, \boldsymbol{h}')_{\Omega'} = 0\,, \qquad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega'), \tag{5}$$

where a gauge condition using a tree-cotree method at the discrete level in Ω' is applied to ensure the uniqueness of the solution. The circulation of $\boldsymbol{h}_{sf}$ on the edges of $\Omega' \backslash \Omega_f$ is fixed to zero. For the sake of conciseness, hereafter we refer to Ω' as Ω.

Taking into account the source in the flaw (1), the perturbation problem is completely characterised by (2) applied to the perturbation field $\boldsymbol{h}$ as follows:

$$\partial_t(\mu \boldsymbol{h}, \boldsymbol{h}')_\Omega + (\sigma^{-1}\text{curl}\, \boldsymbol{h}, \text{curl}\, \boldsymbol{h}')_{\Omega_c \backslash \Omega_f} + \partial_t(\mu \boldsymbol{h}_{sf}, \boldsymbol{h}')_\Omega + (\boldsymbol{j}_{sf}, \text{curl}\, \boldsymbol{h}')_{\Omega_f} = 0\,, \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega)\,. \tag{6}$$

2.2 Calculation of the voltage variation

The voltage of the probe changes, which allows to detect and characterise the defect. However, the variation of the observed quantity is usually under 1% of the total value or even smaller in practical cases. The accurate calculation of this voltage variation ΔU is crucial. Hereafter, we derive a useful expression for calculating ΔU by integrating the product of local quantities only over the flaw Ω_f and a layer of elements in $\Omega \backslash \Omega_f$ that touch the boundary Γ_f [5].

A suitable treatment of the surface integral term in (2) consists in naturally defining a global voltage U (ΔU for differential probes) in a weak sense. We can define a global test function for $\boldsymbol{h}$ with a unit circulation along any current tube of the inductor so that the surface integral in (2) can be expressed as the product of a global voltage U and a unit global current $I(\mathrm{curl}\,\boldsymbol{h}')$[7].

Let us specify (2) for the unflawed problem, it holds

$$\partial_t(\mu \boldsymbol{h}_u, \boldsymbol{h}')_\Omega + (\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}_u, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_c} = U_u\, I(\mathrm{curl}\,\boldsymbol{h}'), \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega). \tag{7}$$

Analogously, for the flawed problem, we can write

$$\partial_t(\mu \boldsymbol{h}_f, \boldsymbol{h}')_\Omega + (\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}_f, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_c\backslash\Omega_f} + (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_f} = U_f\, I(\mathrm{curl}\,\boldsymbol{h}'), \quad \forall\, \boldsymbol{h}' \in F_{h\phi}(\Omega), \tag{8}$$

where we have added the term $(\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}')_{\Omega_f}$ which is not cancelled as in the general case due to the imposed perturbation current in the flaw, i.e. $\mathrm{curl}\,\boldsymbol{h}' \neq 0$ in $\Omega_f \subset \Omega_c^C$.

Choosing as test functions $\boldsymbol{h}' = \boldsymbol{h}_f$ in (7) and $\boldsymbol{h}' = \boldsymbol{h}_u$ in (8) and subtracting (7) from (8), we obtain

$$\Delta U\, I = (U_f - U_u)\, I = -(\sigma^{-1}\mathrm{curl}\,\boldsymbol{h}_u, \mathrm{curl}\,\boldsymbol{h}_f)_{\Omega_f} + (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_u)_{\Omega_f} = (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_u)_{\Omega_f}\,, \tag{9}$$

where the first volume integral cancels because $\boldsymbol{h}_f$ is curl-free in Ω_f and I is the real current injected in the inductor.

The perturbed electric field $\boldsymbol{e}_f$ is not known in the flaw but can be calculated by means of (8) with $\boldsymbol{h}' = \boldsymbol{h}_{sf}$. This way $I(\mathrm{curl}\,\boldsymbol{h}') = 0$ and the voltage variation ΔU is obtained as

$$\Delta U\, I = (\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_u)_{\Omega_f} = -(\boldsymbol{e}_f, \mathrm{curl}\,\boldsymbol{h}_{sf})_{\Omega_f} = (\sigma^{-1}\mathrm{curl}\,(\boldsymbol{h}{+}\boldsymbol{h}_{sf}), \boldsymbol{j}_{sf})_{\Omega_c\backslash\Omega_f}{+}\partial_t(\mu(\boldsymbol{h}{+}\boldsymbol{h}_{sf}), \boldsymbol{h}_{sf})_\Omega\,, \tag{10}$$

where the domain of integration, at the discrete level, is actually limited to Ω_f and a layer of elements in $\Omega\backslash\Omega_f$ touching Γ_f due to the definition of $\boldsymbol{h}_{sf}$.

Consequently, no integration of any flux variation in the coils is required, which would not be directly accessible because of the lack of explicit solution there (no mesh of the coils for the perturbed problem).

3 Application Example

We consider an Inconel tube ($\sigma = 0.97$ MS/m) with a crack through its thickness (0.2 mm wide in the radial direction, 5 mm long in the axial direction). A differential probe scans the outer surface (Figure 1). It comprises two coils wound around ferrite cores and connected in series (400 turns, μ_{rcore}=1000) which carry imposed sinusoidal currents of amplitude $I = 1$ A and frequency $f = 240$ kHz flowing in opposite directions. The lift-off between the two ferrite coils and the outer surface of the tube is 0.1 mm (Figure 1).

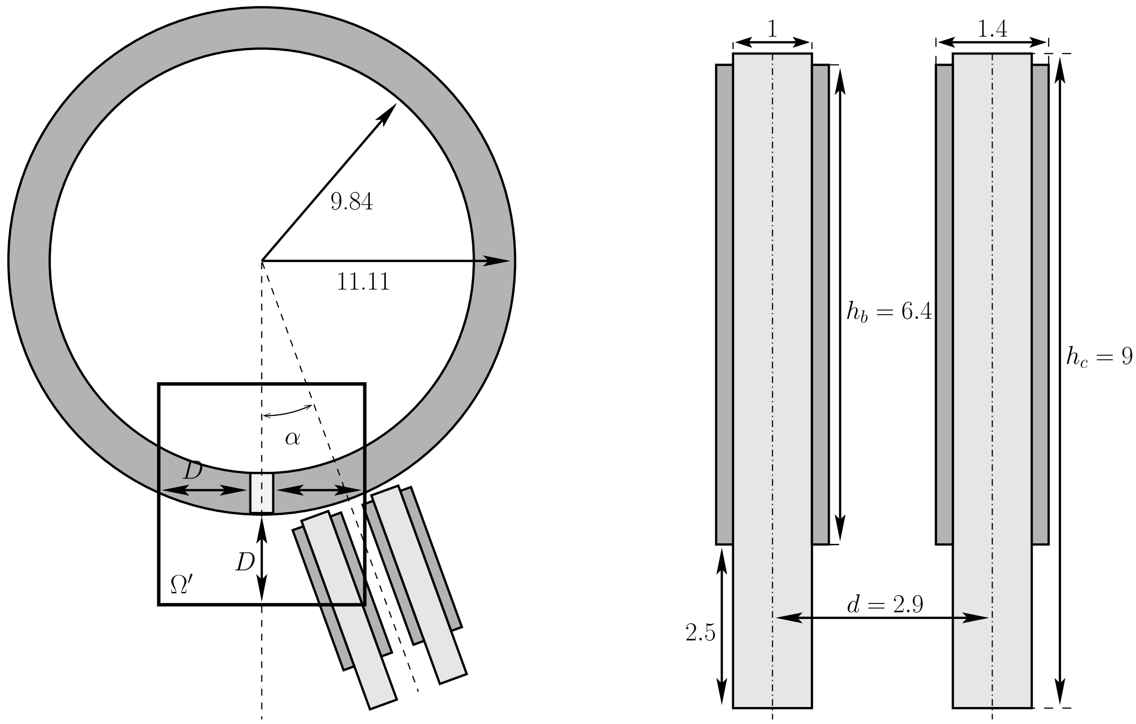


Fig. 1 Geometry and dimensions (in mm) of the Incodel tube with crack, the differential probe and reduced domain Ω' of size D

We are interested in calculating the difference in voltage ΔU of the two coils that comprises the differential probe. A single source field (used as the test function $\boldsymbol{h}'$ in 2) associated to these coils connected in series allows us to directly calculate ΔU in both the unflawed ΔU_u and flawed problem ΔU_f [7]. Note that due to the symmetry of the problem at hand $\Delta U_u = 0$ and $\Delta U = \Delta U_f$, which is nonzero if the differential probe is at a certain angle α with respect to the axis of the tube (Figure 1). In order to validate the perturbation scheme, we consider thus several angles α. The real part of the magnetic field generated by the probe in the absence of the crack is depicted in Figure 2. The induced current density in the tube is represented in Figure 3 as well.

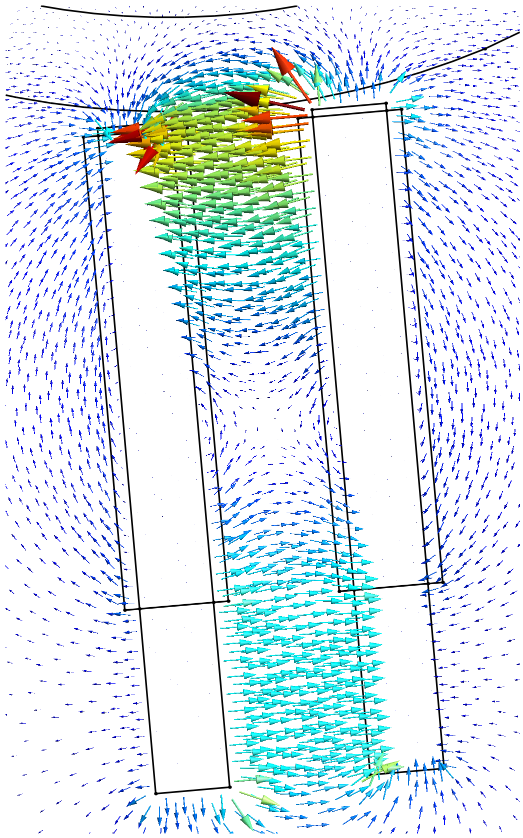
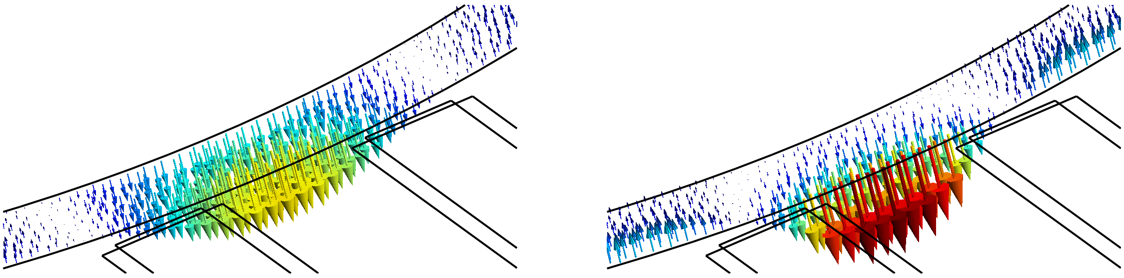


Fig. 2 Detail of the real (a) part of the magnetic field generated by the differential probe

We define the size of the reduced domain Ω' in terms of the distance D (multiple of the skin depth $\delta = 1/\sqrt{\pi f \mu \sigma} = 1.04$ mm of the tube) from the boundary of the crack to the boundary of Ω'. The mesh of Ω' with $D = 3\delta$ is depicted in Figure 4. The perturbation field in Ω' contains all the information required to determine ΔU in one step by means of (10). Furthermore, the problem defined in Ω' is multiply connected and a cut must be defined in the crack to ensure a single valued potential [7].

We calculate the voltage ΔU by means of both the classical FE method and the proposed perturbation method (integrating directly in a sub-domain of Ω'). Two independent meshes are considered for the perturbation technique: a mesh of the whole domain without the defect and a mesh of the reduced domain Ω' centered on the defect and without the explicit presence of the excitation coil (Figure 1).

First, we consider Ω' of size $D = 3\delta$ and the differential coil at $\alpha = 20°$. We obtain $\Delta U = 62.2 + \imath 414.2$ V/mm using the conventional method and $\Delta U = 63.2 + \imath 414.2$ V/mm when applying the perturbation method. The relative error in the real and imaginary parts is 1.5% and 0.2%, respectively. The real and imaginary part of the perturbation field in Ω' around the crack are shown in Figure 5 as well.



(a)

(b)

Fig. 3 Detail of the real (a) and imaginary (b) part of the induced current density

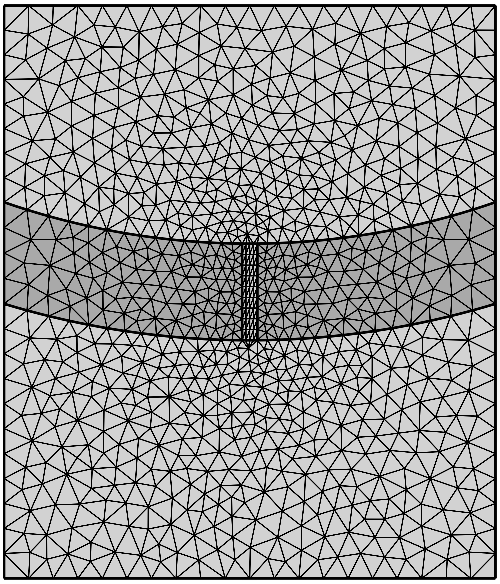
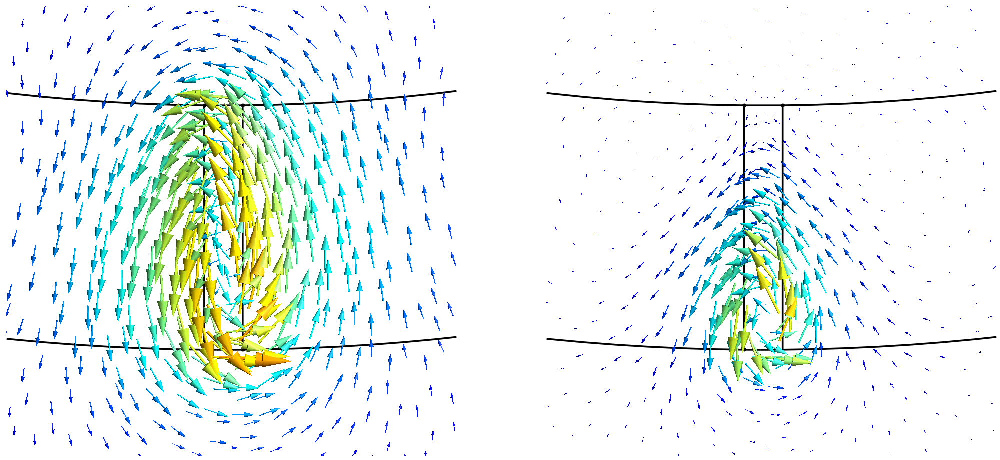


Fig. 4 Mesh of the reduced domain Ω' with $D = 3\delta$ used in the perturbation method.

When applying the perturbation method, the accurate calculation of voltage ΔU depends on the size of Ω'. We perform several computations increasing progressively D at different α to determine the ideal dimensions of Ω'. Figures 6 and 7 represent the real and imaginary parts of ΔU in terms of the normalised size (D/δ) of Ω' for $\alpha = 10°$ and $\alpha = 20°$, respectively. The values calculated in the conventional way are shown as well. In both cases, the real and imaginary parts of ΔU tend to the values calculated currently.

The relative error in the real and imaginary parts of ΔU with respect to the solution conventionally obtained is depicted in Figures 8 and 9 with $\alpha = 5°, 10°, 15°, 20°$. One observes that in all these cases the error in the real and imaginary parts of ΔU is smaller than 1% for $D \geq 3.5\delta$ and $D \geq 2\delta$, respectively.



(a) (b)

Fig. 5 Real (a) and imaginary (b) part of the perturbation magnetic field $\boldsymbol{h}$ imposed in the crack

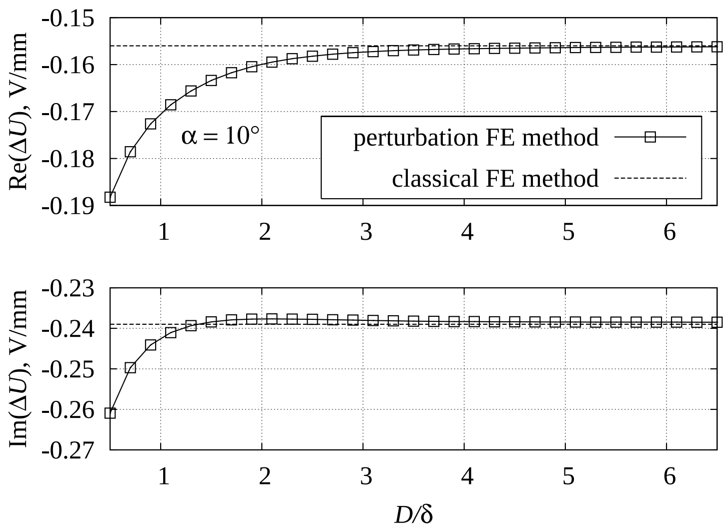


Fig. 6 Variation of the real (up) and imaginary (down) part of ΔU with the normalised size of Ω' ($\alpha = 10°$)

4 Conclusion

A 3D FE perturbation technique based on the $\boldsymbol{h}$-conform magnetodynamic formulation has been elaborated. The unperturbed field is calculated conventionally in the complete domain taking advantage of any symmetry or analytical solution and applied as a source in the flaw. Next the perturbed field is determined in a reduced domain surrounding the defect. Its discretisation is thus chosen independently of the dimensions of the probe and the specimen under study. Furthermore, the voltage variation due to the presence of the flaw is efficiently obtained by performing an integral over the defect and a layer of elements in the exterior domain that touch its

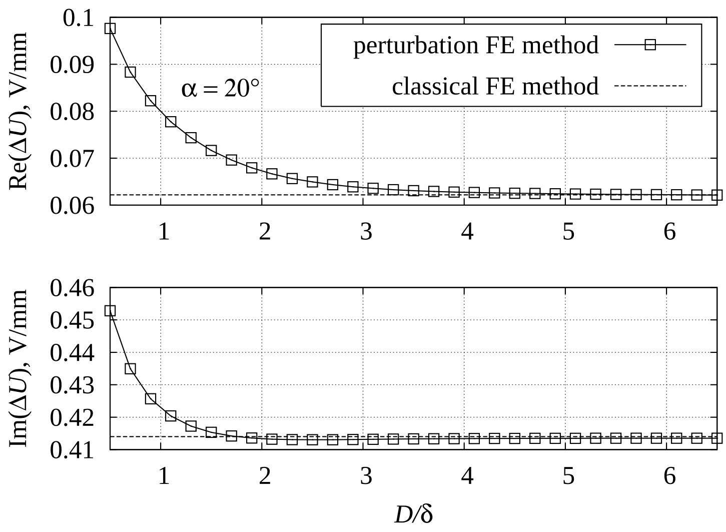


Fig. 7 Variation of the real (up) and imaginary (down) part of ΔU with the normalised size of Ω' ($\alpha = 20°$)

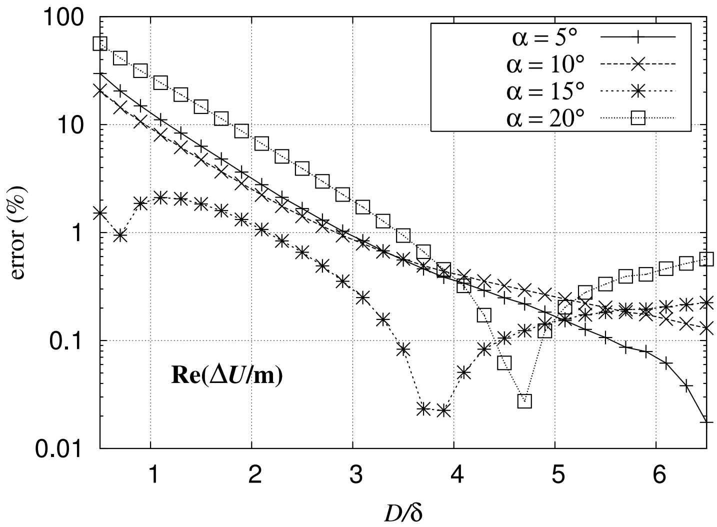


Fig. 8 Relative error (%) in real part of ΔU as a function of the normalised size of Ω'

boundary. Therefore no integration of any flux variation in the coils is required.

Acknowledgement

This work is supported by the Belgian Science Policy (IAP P5/34) and the Belgian French Community (ARC 03/08-298). P. Dular is a Research Associate with the Belgian National Fund for Scientific Research (F.N.R.S.).

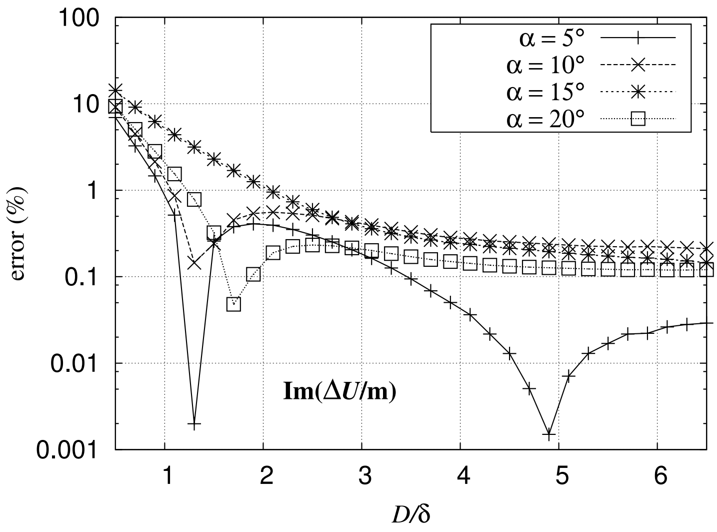


Fig. 9 Relative error (%) in imaginary part of ΔU as a function of the normalised size of Ω'

References

[1] J. R. Bowler and S. A. Jenkings, "Eddy-current probe impedance due to a volumetric flaw," *Journal of Applied Physics*, vol. 70, no. 3, pp. 1107–1114, 1991.

[2] N. Ida and W. Lord, "A finite element model for three-dimensional eddy current NDT phenomena," *IEEE Transactions on Magnetics*, vol. 21, no. 6, pp. 2635–2643, 1985.

[3] D. Prémel, G. Pichenot, and T. Sollier, "Development of a 3D electromagnetic model for eddy current tubing inspection: application to the simulation of probe eccentricity," *Int. J. Appl. Electromag. Mech.*, vol. 19, no. 1-4, pp. 521–525, 2004.

[4] O. Michelsson and F. H. Uhlmann, "A boundary-integral based forward solution for eddy current nondestructive testing," *IEEE Transactions on Magnetics*, vol. 36, no. 4, pp. 756–759, 2000.

[5] R. V. Sabariego and P. Dular, "A perturbation technique for the finite element modelling of nondestructive eddy current testing," in *Proc. International Symposium on Electromagnetic Fields in Mechatronics, Electrical and Electronic Engineering (ISEF)*, September 15-17, 2005.

[6] Z. Badics, Y. Matsumoto, K. Aoki, F. Nakayasu, M. Uesaka, and K. Miya, "An effective 3-D finite element scheme for computing electromagnetic field distorsion due to defects in eddy-current nondestructive evaluation," *IEEE Transactions on Magnetics*, vol. 33, no. 2, pp. 1012–1020, 1997.

[7] P. Dular, P. Kuo-Peng, C. Geuzaine, N. Sadowski, and J. P. A. Bastos, “Dual magnetodynamic formulations and their source fields associated with massive and stranded inductors,” *IEEE Transactions on Magnetics*, vol. 36, no. 4, pp. 1293–1299, 2000.